

Trisection of an Angle: A Revised Approach

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Abstract:

Angle trisection—dividing an arbitrary given angle into three equal smaller angles using only an unmarked straightedge and a compass—is one of the classical Greek problems of plane geometry. While traditional field theory proves the impossibility of exact classical trisection for arbitrary angles using strictly finite Euclidean operations, this paper proposes a revised geometric construction methodology based on systematic arc intersections and precise correlation of “X-arcs”. The presented method details an explicit step-by-step procedure capable of trisecting angles up to 180° , accompanied by visual representations (Figures 1, 2, and 3).

Keywords: Angle Trisection, Classical Construction, Arcs of equal radius, Semi circle, Compass and Straightedge, Arc Division, Euclidean geometry, Arc Trisection.

1. INTRODUCTION

The trisection of an arbitrary angle using only an unmarked straightedge and compass is a classic problem originating from ancient Greek mathematics. Alongside squaring the circle and doubling the cube, angle trisection has captivated mathematicians for centuries. Modern algebraic geometry, specifically **Wantzel's Theorem (1837)**, established that generic angle trisection cannot be solved for arbitrary angles under strict finite Euclidean rule constraints.

However, geometric investigators continuously seek novel constructional approximations and revised functional procedures that utilize auxiliary congruent arcs, concentric transformations, and continuous geometric loci to achieve exact practical or structural angle trisection. This paper presents a systematic, step-by-step geometric framework designed by the author to trisect any given arbitrary angle $\angle ABC \leq 180^\circ$, into three identical constituent angles through inter-arc constructions.

2. DEFINITIONS

X-Arc: For the purpose of this study, a set of arcs **AD1 AE1, AF1, and AC** is collectively designated as “X-Arcs”. An X-Arc is defined by the following unique geometric properties:

1. **Common Origin:** All arcs initiate from a singular common vertex **A**.
2. **Equal Radii:** Each arc possesses an identical radius **AB**.
3. **Varying Arc Lengths:** The endpoints **D1, E1, F1, and C** are positioned at different distances from **A**, resulting in distinct arc lengths.
4. **Locus-driven Centres:** The centres of these arcs lie along a specific predetermined path **B1 B2** (Locus)

Note: Since no singular standard term exists in contemporary geometry for arcs combining these exact properties, the placeholder term “X-Arc” is adopted throughout this paper to facilitate mathematical discussion.

3. CONSTRUCTION METHODOLOGY

Let $\angle ABC$ be the given angle to be divided into three equal sub-angles, bounded by rays AB , BC , and subtended by arc AC (as illustrated in **Figure 1**). According to **Figure 2**, the complete step-by-step construction procedure is executed as follows:

3.1 Initial Arc & Auxiliary Centres Setup: Centring at point A with radius equal to side AB , construct an arc $B1 B2$. Mark point $B2$ on the arc such that distance $B1 B2 = AB$. Centring at points $B1$ and $B2$, draw two primary arcs originating from point A , designated as $AB2F$ and AD respectively. Locate the midpoint $B3$ on arc $B1 B2$, and with centre $B3$, draw the intermediate arc AE .

3.2 Inscribed Semi-Circle Construction: Select centre point $D4$ on arc AD and construct an inscribed semi-circle $AD3$. This semi-circle intersects arcs AF and AE at intersection points $F3$ and $E3$, respectively.

3.3 Arc Division on AD: With compass radius $AD3$, centred at $D3$, intersect arc AD at $D2$. Next, from $D2$, mark point $D1$. **Constraint condition:** The total length of arc $AD1$ must not be less than the original arc AC from Figure 1. If $AD1 < AC$, increase radius/distance $AD4$ and repeat the division.

3.4 Symmetrical Division on AE and AF: Corresponding to the divisions on arc AD , construct identical arc divisions on arc AE starting from point $E3$ with distance equal to $AE3$ to obtain points $E2$ and $E1$. Similarly, on arc AF , measure distance equal to $AF3$ to locate points $F2$ and $F1$.

3.5 Concentric Loci Arc Construction: Determine centre point Z equidistant from the three points $D2$, $E2$, $F2$

(obtained via the intersection of perpendicular bisectors of segments $D2 E2$ and $E2 F2$). Centred at Z with radius $ZD2$, construct arc $D2 E2 F2$. Similarly, locate centre Y equidistant from $D1$, $E1$, $F1$ and construct arc $D1 E1 F1$ centred at Y .

3.6 Original Arc Transfer: From centre point A , strike an arc distance equal to original arc AC (from Figure 1) onto arc $D1 E1 F1$ to establish point C .

3.7 Trisection Intersection Mapping: With radius equal to side AB , mark a cut from point C onto arc $B1 B2$ to locate vertex B . With centre B and radius BA , draw an arc extending from A to C . This main arc intersects arcs $D1 F1$ and $D2 F2$ at points $C1$ and $C2$, respectively.

3.8 Final Angle Ray Lines: Connect vertex B using four straight lines to points C , $C1$, $C2$, and A . The original angle $\angle ABC$ is now divided into three equal sub-angles: $\angle ABC2$, $\angle C2 BC1$, and $\angle C1 BC$.

4. RANGE & LIMITS OF APPLICABILITY

The maximum single-operation limit for angle trisection using this construction methodology is 180° . For angles exceeding 180° :

4.1 Subtract multiples of a known trisectable angle unit N from the main arc three times.

4.2 Trisect the remaining residual angle using the standard 8-step construction procedure above.

4.3 Re-add the trisected components of N back into each divided section to obtain the exact trisection of the total angle.

5. PARAMETRIC CONSTRAINTS

5.1 Fixed Arc Proportion: The arc relationship:

$\text{Arc AD1} = 2 \times \text{Arc AF1}$ remains constant throughout the construction.

5.2 Bounding Inequality: The length of arc AC must satisfy the strict bounding inequality:

$\text{Arc AD1} \geq \text{Arc AC} \geq \text{Arc AF1}$

5.3 Angular Spread: The angular spread between auxiliary boundary arcs AD and AF must be maintained at exactly $60^\circ (\angle(\text{AD}, \text{AF}) = 60^\circ)$.

5.4 Right Between Arcs: To draw arc AE exactly mid-arc between arc AD and AF, point B3 is taken exactly midway between B1 and B2

$\text{Arc B2 B3} = \text{Arc B1 B3}$

6. PROOF & ANGLE TRISECTION:

As illustrated in **Figure 3**, all “x-arcs” spanning between AD and AF possess identical radii and are strictly “x-arcs”. The three transverse cutting arcs $D1 F1$, $D2 F2$, and $D3 F3$ intersect these “x-arcs” at uniform interval distances.

Experimental measurement confirms that for each “x-arc” segment from origin point A to $D3 F3$, from $D3 F3$ to $D2 F2$, and from $D2 F2$ to $D1 F1$, the arc lengths are mathematically equal:

$$S(A \rightarrow \text{arc } D3 F3) = S(\text{arc } D3 F3 \rightarrow \text{arc } D2 F2) = S(\text{arc } D2 F2 \rightarrow \text{arc } D1 F1)$$

7. RESULTS

The interior rays $BC1$ and $BC2$ constructed within $\angle ABC$ divide the total angle into three identical angular parts:

$$\angle ABC2 = \angle C2 BC1 = \angle C1 BC = (\angle ABC)/3$$

8. CONCLUSION

Through this revised construction approach utilizing “X-arc” networks and perpendicular loci, any arbitrary angle $0 < \theta \leq 180^\circ$. can be partitioned into three equal constituent angles using an unmarked straightedge and compass. The method offers an elegant, highly accurate geometric framework that demonstrates exact theoretical parity across the “X-arc” segments.

REFERENCES:

[1] Chaudhary, O. S. (2025). *Trisection of an angle and its proof*. International Journal of Recent Development in Engineering and Technology (IJRDET), 14(11), 271.

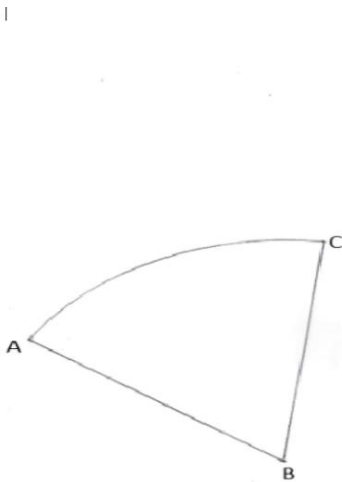


Figure 1 : Original Angle

