

Some Properties of Generalized Contra fuzzy lattice Operator group

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Abstract:

In this paper a generalized structure of contra fuzzy lattice operator group is defined. In this structure n operators are used which are members of n different sets called operator sets. Also in this paper Some properties of this structure are derived.

Keywords: Lattice group, Contra Fuzzy lattice group, Contra Fuzzy lattice KS operator group

Introduction

A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set $[0,1]$ by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagarajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of Contra fuzzy lattice KS operator group and some of its properties.

1. PRELIMINARIES

Definition 1.1 Contra Fuzzy lattice KS- operator group (CFL KS- operator group)

$\lambda: X$ to $[0, 1]$ be a fuzzy set, Let G be a subset of X which is a lattice KS- operator group , K, S (operator sets). λ is a function over G . It is a Contra fuzzy lattice KS- operator group if it satisfy following four conditions

- 1) $\lambda(kx sy) \leq \max\{\lambda(kx), \lambda(sy)\}$
- 2) $\lambda(kx)^{-1} \leq \lambda(kx) \ \& \ \lambda(sx)^{-1} \leq \lambda(sx)$

- 3) $\lambda(kx \vee sy) \leq \max\{\lambda(kx), \lambda(sy)\}$
- 4) $\lambda(kx \wedge sy) \leq \max\{\lambda(kx), \lambda(sy)\}$ For every $x \in G, k \in K, s \in S$

Definition 1.2 Contra Fuzzy lattice KK -operator group

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice KK-operator group if it satisfy following four conditions

- 1) $\lambda(k_1 x k_2 y) \leq \max\{\lambda(k_1 x), \lambda(k_2 y)\}$
- 2) $\lambda(k_1 x)^{-1} \leq \lambda(k_1 x), \lambda(k_2 x)^{-1} \geq \lambda(k_2 x),$
- 3) $\lambda(k_1 x \vee k_2 y) \leq \max\{\lambda(k_1 x), \lambda(k_2 y)\}$
- 4) $\lambda(k_1 x \wedge k_2 y) \leq \max\{\lambda(k_1 x), \lambda(k_2 y)\},$ For all $x, y \in G, k_1, k_2 \in K$

Definition 1.3 Contra Fuzzy lattice K²-operator group

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice K²-operator group if it satisfy following four conditions

- 1) $\lambda(kxky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 2) $\lambda(kx)^{-1} \leq \lambda(kx)$
- 3) $\lambda(kx \vee ky) \leq \max\{\lambda(kx), \lambda(ky)\}$
- 4) $\lambda(kx \wedge ky) \leq \max\{\lambda(kx), \lambda(ky)\}$ For all $x, y \in G, k \in K$

Definition 1.4 Contra Fuzzy lattice K₁K₂.....K_n -operator group

$\lambda: X$ to $[0, 1]$ is a fuzzy set; G is a K - lattice operator group, A function λ on G is said to be a Contra fuzzy lattice KK-operator group if it satisfy following four conditions

- 1) $\lambda(k_1 x_1 k_2 x_2 \dots k_n x_n) \leq \max\{\lambda(k_1 x_1), \lambda(k_2 x_2), \dots, \lambda(k_n x_n)\}$
 - 2) $\lambda(k_1 x_1)^{-1} \leq \lambda(k_1 x_1), \lambda(k_2 x_2)^{-1} \leq \lambda(k_2 x_2), \dots, \lambda(k_n x_n)^{-1} \leq \lambda(k_n x_n),$
 - 3) $\lambda(k_1 x_1 \vee k_2 x_2 \dots k_n x_n) \leq \max\{\lambda(k_1 x_1), \lambda(k_2 x_2), \dots, \lambda(k_n x_n)\}$
 - 4) $\lambda(k_1 x_1 \wedge k_2 x_2 \dots k_n x_n) \leq \max\{\lambda(k_1 x_1), \lambda(k_2 x_2), \dots, \lambda(k_n x_n)\}$
- , For all $x_1, x_2, x_3, \dots \in G, k_i \in K_i$

Definition 1.5

Let $\lambda: X$ to Y be a function. Q is a fuzzy group of Y . A fuzzy set λ^{-1} Inverse image of Q under λ is given by $\lambda^{-1}(Q) = \mu_{\lambda^{-1}(Q)}(x) = \mu_Q \lambda(x)$

Definition 1.6

$\mu_A: X$ to $[0, 1]$ be a fuzzy set and $\lambda: X \rightarrow X'$ is a function. A function $\mu_{A\lambda}: X$ to $[0, 1]$ is defined by $\mu_{A\lambda}(x) = \mu_A \lambda(x)$

Definition 1.7

If T and T' are lattice KS- operator groups .A function

$\lambda: T$ to T' be a lattice KS homomorphism

if $\lambda(kxsy) = \lambda(kx)\lambda(sy) = k\lambda(x)s\lambda(y), \lambda(kx \vee sy) = \lambda(kx) \vee \lambda(sy) = k\lambda(x) \vee s\lambda(y),$
 $\lambda(kx \wedge sy) = \lambda(kx) \wedge \lambda(sy) = k\lambda(x) \wedge s\lambda(y)$ For all $x, y \in G, k \in K, s \in S$

Definition 1.8

Let A_i be a contra fuzzy lattice KS operator groups of G_i , for $i = 1, 2, \dots, n$. Then the product A_i ($i = 1, 2, \dots, n$) is the function $A_1 \times A_2 \times \dots \times A_n: G_1 \times G_2 \times \dots \times G_n \rightarrow [0, 1]$ defined by

$$(A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n) = \max\{A_1(x_1), A_2(x_2), \dots, A_n(x_n)\}$$

2. PROPERTIES OF CONTRA FL KS- operator GROUP

Proposition 2.1: Let T and T' be two Lattice $K_1 K_2 \dots K_n$ operator groups and $\lambda: T \rightarrow T'$ be a lattice $K_1 K_2 \dots K_n$ homomorphism. If P is a Contra FL $K_1 K_2 \dots K_n$ operator group of T then the fuzzy set $P^\lambda = \{x \in T; \mu_{P^\lambda}(x) = \mu_P(\lambda(x)), x \in T\}$ is Contra FL KS operator group of T .

Proposition 2.2: If $\lambda: T \rightarrow T'$ be a surjective lattice $K_1 K_2 \dots K_n$ homomorphism and let P be a Contra fuzzy lattice $K_1 K_2 \dots K_n$ operator group of T . Define a fuzzy set $P^\lambda: T' \rightarrow [0, 1]$ by $P^\lambda(x') = \max\{P(x)/x \in \lambda^{-1}(x')\}$. Then P^λ is a Contra fuzzy lattice $K_1 K_2 \dots K_n$ operator group on T' .

Proposition 2.3: Let $\lambda: T \rightarrow T'$ be a lattice $K_1 K_2 \dots K_n$ homomorphism and P' be a Contra fuzzy lattice KS operator group of T' then $\lambda^{-1}(P')$ is a Contra fuzzy lattice $K_1 K_2 \dots K_n$ operator group of T .

Proposition 2.4: Direct product of Contra fuzzy lattice $K_1 K_2 \dots K_n$ operator groups is also a Contra fuzzy lattice $K_1 K_2 \dots K_n$ operator group.

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