International Journal of Leading Research Publication (IJLRP)

E-ISSN: 2582-8010 ● Website: www.ijlrp.com ● Email: editor@ijlrp.com

Comparative Study of NSGA-II, MOEA/D, and SPEA2 in Fuzzy Multi-Objective Optimization

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Abstract:
This study presents a comprehensive comparative analysis of three leading evolutionary algorithms—NSGA-II, MOEA/D, and SPEA2—within a fuzzy multi-objective optimization framework. The primary goal is to evaluate their performance in solving complex optimization problems under uncertainty, where objectives and constraints are represented through fuzzy sets. Benchmark problems from the ZDT and DTLZ families were employed to assess each algorithm's efficiency based on convergence, diversity, and robustness metrics such as Hypervolume (HV), Generational Distance (GD), Inverted GD (IGD), Spread, and Spacing. Experimental results reveal that the integration of fuzzy modeling significantly enhances optimization performance by providing flexibility and robustness against imprecise or uncertain data. Among the algorithms compared, the fuzzy-enhanced NSGA-II demonstrated superior convergence to the Pareto front, higher diversity, and improved stability, followed by MOEA/D, while SPEA2 showed comparatively lower performance. Statistical tests confirmed the significance of these results, establishing that fuzzy-based multi-objective optimization can yield more realistic and reliable decision outcomes in uncertain environments.

Keywords: NSGA-II, MOEA/D, SPEA2, Fuzzy Multi-Objective Optimization, Evolutionary Algorithms, Pareto Front, Hypervolume, Generational Distance, Spread, Uncertainty Modeling.

1. INTRODUCTION

Multi-objective optimization (MOO) has become an indispensable area of research in engineering, operations research, and artificial intelligence, given the prevalence of problems requiring trade-offs among conflicting objectives. Real-world systems, especially in areas like manufacturing, transportation, environmental modeling, and smart infrastructure, demand solutions that balance performance, cost, reliability, and environmental impact simultaneously. Classical optimization techniques struggle to handle such problems due to their reliance on scalarized formulations and deterministic procedures. As a result, evolutionary multi-objective optimization (EMO) algorithms have emerged as promising alternatives due to their flexibility, population-based search strategies, and ability to approximate Pareto-optimal solutions in a single run (Deb, 2011; Coello Coello & Lamont, 2004).

Among the most prominent EMO algorithms are Non-dominated Sorting Genetic Algorithm II (NSGA-II), Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D), and Strength Pareto Evolutionary Algorithm 2 (SPEA2). These algorithms differ in terms of Pareto sorting, diversity preservation, and convergence mechanisms. NSGA-II relies on non-dominated sorting and crowding distance to maintain a diverse front (Deb et al., 2002). MOEA/D decomposes the MOO problem into scalar sub-problems and solves them cooperatively (Zhang & Li, 2007). SPEA2 assigns strength-based fitness and uses an external archive for elitism and density estimation (Zitzler et al., 2001). These algorithms have proven effective across a wide range of applications, yet comparative performance varies significantly depending on the problem landscape, objective dimensionality, and presence of constraints (Zitzler et al., 2000; Wu & Zhou, 2016).

A particularly challenging extension of MOO is when uncertainty and imprecision are present in problem definitions, constraints, or objective evaluations. In real-world decision-making, especially in systems involving human preferences, sensor noise, or linguistic modeling, crisp representations are often inadequate. To address this, fuzzy logic has been integrated with evolutionary algorithms, giving rise to Fuzzy Multi-Objective Optimization (FMOO) techniques (Herrera & Lozano, 2003). Fuzzy systems allow the modeling of vague inputs using membership functions and linguistic variables. In FMOO, objectives can be fuzzy, preferences can be described using fuzzy rules, and even dominance relations can be fuzzified. This leads to more robust, adaptable solutions under deep uncertainty (Chanda & Pal, 2018; Jin & Sendhoff, 2008).

The integration of fuzzy logic into EMO algorithms has spawned multiple fuzzy-enhanced variants of NSGA-II, MOEA/D, and SPEA2. For instance, fuzzy NSGA-II variants incorporate fuzzy dominance or use fuzzy rule bases to guide the search (Li et al., 2015). Fuzzy MOEA/D incorporates fuzzy decomposition of objectives and fuzzy neighborhood structures to adapt better to noisy or vague environments (Zhang et al., 2010). Similarly, fuzzy SPEA2 uses fuzzy fitness metrics and density estimations to maintain solution diversity and robustness (Chanda & Pal, 2018). These fuzzy-EMO hybrids have been applied in scheduling, control system design, energy optimization, and resource allocation under uncertainty (Pan et al., 2010; Yang et al., 2017).

Despite the existence of these fuzzy variants, a comprehensive comparative evaluation of NSGA-II, MOEA/D, and SPEA2 within fuzzy MOO environments is still lacking in literature. While individual studies have benchmarked fuzzy NSGA-II or fuzzy MOEA/D on synthetic problems, few studies have undertaken a systematic and statistically validated comparison across standardized test functions and performance metrics. This gap is significant given the growing importance of robust optimization in the context of Industry 4.0, smart systems, and uncertain cyber-physical environments (Wu & Zhou, 2016; Sinha et al., 2014).

The purpose of this study is to conduct a comparative evaluation of NSGA-II, MOEA/D, and SPEA2 when applied in a fuzzy multi-objective optimization context. The study leverages benchmark test problems like ZDT, DTLZ, and real-world inspired problems under fuzzy representations of objectives. The performance of each algorithm is assessed based on convergence (Generational Distance, Epsilon Indicator), diversity (Spacing, Spread), and robustness under uncertainty (Hypervolume under noise). Statistical tests such as ANOVA and Wilcoxon rank-sum are used to assess the significance of performance differences. In doing so, the study builds upon prior foundational work (Zitzler et al., 2000; Jin, 2005; Deb et al., 2002) while extending the methodology to incorporate fuzzy systems and robustness evaluation.

The study is grounded in the broader theoretical developments in fuzzy set theory and its synergy with evolutionary algorithms. Type-1 and Interval Type-2 Fuzzy Sets (IT2FS) have gained prominence in modeling uncertainty, particularly where epistemic and aleatory uncertainties coexist (Herrera & Lozano, 2003). In FMOO, fuzzy rule-based systems are used to define the relationship between input and output variables, enabling the algorithms to navigate noisy and linguistically defined search spaces. For example, fuzzy sensitivity analysis has been employed to evaluate the impact of design variables on multiple objectives (Chanda & Pal, 2018), while fuzzy surrogates are used to reduce computational cost in expensive simulations (Jin, 2005).

Furthermore, the study aligns with the recent shift toward interpretable and explainable optimization. With the rise of black-box models in AI and control systems, the need for optimization frameworks that provide not only accurate but interpretable Pareto-optimal solutions is crucial (Jin & Sendhoff, 2008). Fuzzy logic supports this requirement through rule-based systems and linguistic output that can be interpreted by human decision-makers.

This paper contributes to the literature in several ways. First, it provides a unified experimental platform to compare NSGA-II, MOEA/D, and SPEA2 using fuzzy-enhanced representations. Second, it introduces a comprehensive evaluation framework that includes robustness to uncertainty, convergence metrics, and diversity measures. Third, the study provides insights into the relative strengths and weaknesses of each algorithm under different fuzzy scenarios, guiding future researchers and practitioners in algorithm selection. Finally, it lays the groundwork for future hybrid approaches that may combine the best elements of all three algorithms with deep fuzzy logic and learning-based enhancements (Yang et al., 2017; Sinha et al., 2014).

2. LITERATURE REVIEW

Multi-Objective Optimization and Metaheuristics

Multi-objective optimization (MOO) addresses problems involving multiple conflicting objectives, common in complex real-world systems like control engineering, machine learning, and resource planning. Traditional mathematical approaches often fall short in solving such problems due to high dimensionality, non-linearity, and uncertainty. In response, evolutionary algorithms (EAs) have emerged as powerful tools for solving MOO problems without requiring gradient information (Deb et al., 2002; Deb, 2011). These algorithms work on populations of solutions and employ stochastic operators to explore the search space efficiently.

Among the most influential MOO algorithms are the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb et al., 2002), the Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) (Zhang & Li, 2007), and the Strength Pareto Evolutionary Algorithm 2 (SPEA2) (Zitzler et al., 2001). Each of these has distinct mechanisms for handling Pareto dominance, solution diversity, and selection pressure.

NSGA-II: Baseline for Fuzzy MOO

NSGA-II has been widely used as a benchmark algorithm in MOO research due to its elitist selection, fast non-dominated sorting, and crowding distance metrics (Deb et al., 2002). It has been successfully applied to problems involving uncertainty and noise, especially in systems requiring fuzzy modeling. Deb's (2011) comprehensive work further detailed its real-world application potential.

However, NSGA-II does not natively incorporate mechanisms to deal with epistemic or aleatory uncertainty. To address this, researchers have extended it by integrating fuzzy systems, resulting in Fuzzy-NSGA-II algorithms capable of evaluating fitness functions using linguistic and uncertain variables (Li et al., 2015; Herrera & Lozano, 2003). These modifications enable better solution quality in uncertain environments.

MOEA/D: Decomposition for Structured Search

MOEA/D is structurally different from NSGA-II. Instead of Pareto dominance, it decomposes the MOO problem into scalar sub-problems that are solved simultaneously using neighborhood-based optimization (Zhang & Li, 2007). This allows better convergence to complex Pareto fronts and is particularly useful in high-dimensional objective spaces. Zhang et al. (2010) introduced adaptive weight update mechanisms in MOEA/D to improve flexibility.

MOEA/D has also been extended with fuzzy systems to enhance decision-making under imprecise input conditions. Its ability to decompose fuzzy preference models allows fine-grained control over decision boundaries, making it suitable for fuzzy MOO in engineering design (Wu & Zhou, 2016).

SPEA2: Strength-Based Pareto Sorting

SPEA2 introduces a fitness assignment strategy based on strength values, along with an external archive and density estimation (Zitzler et al., 2001). It improves over its predecessor (SPEA) by enhancing elitism and maintaining better diversity in the solution set. It has been employed successfully in domains such as flowshop scheduling, control system design, and fuzzy rule generation (Ishibuchi & Murata, 1998; Talbi, 2009).

When integrated with fuzzy logic, SPEA2 demonstrates the ability to handle vague preferences and linguistic fitness criteria, although it sometimes struggles with maintaining convergence in high-noise environments (Chanda & Pal, 2018). Its archive-based approach makes it particularly effective in capturing solution histories for decision-makers.

Fuzzy Systems in Evolutionary Multi-Objective Optimization

Fuzzy logic provides a mathematical structure for dealing with vague, uncertain, and imprecise information. In MOO, fuzzy systems are especially valuable when the objectives or constraints cannot be precisely quantified. Interval Type-2 Fuzzy Sets (IT2FS), for instance, are more expressive than traditional Type-1 fuzzy sets, offering an additional degree of uncertainty modeling (Herrera & Lozano, 2003; Chanda & Pal, 2018).

Fuzzy multi-objective optimization (FMOO) incorporates fuzzy dominance and linguistic variables in both objectives and constraints. Jin (2005) highlighted the role of fuzzy surrogates in reducing computational cost. NSGA-II, MOEA/D, and SPEA2 have all been adapted with fuzzy logic mechanisms, enabling them to handle vagueness in control parameters, performance objectives, and real-world constraints (Li et al., 2015; Jin & Sendhoff, 2008).

Benchmarking and Comparative Studies

Comparative studies among NSGA-II, MOEA/D, and SPEA2 have been crucial in identifying strengths and limitations under different scenarios. Zitzler et al. (2000) demonstrated the empirical performance of early multi-objective algorithms across ZDT and DTLZ test problems, setting a precedent for algorithm benchmarking. More recent studies like Wu and Zhou (2016) have compared MOEA/D and NSGA-II on constrained fuzzy MOO tasks, revealing that decomposition-based methods often outperform Pareto-based ones in high-dimensional settings.

Li et al. (2015) evaluated various fuzzy-integrated EAs across multiple benchmark problems and showed that no single algorithm consistently outperforms others—performance varies depending on the shape of the Pareto front, uncertainty type, and computational budget.

Application Domains and Challenges

The integration of fuzzy logic into MOO has broad applications in control systems, renewable energy optimization, smart manufacturing, and transportation systems (Pan et al., 2010; Yang et al., 2017). These domains are often governed by uncertain variables such as load fluctuations, environmental noise, and human behavioral unpredictability—making fuzzy-EAs a natural fit.

However, one challenge in applying fuzzy logic with EAs is the added computational complexity. Surrogate models and adaptive fuzzy controllers have been proposed to reduce this load (Jin, 2005). Additionally, performance metrics like Hypervolume, Epsilon Indicator, and Generational Distance are often fuzzified to account for tolerance levels in objective functions (Hernández-Díaz et al., 2008).

3. METHODOLOGY

The study employs a simulation-based experimental methodology to systematically compare the performance of NSGA-II, MOEA/D, and SPEA2 within a fuzzy multi-objective optimization framework. Initially, standard benchmark functions from the ZDT and DTLZ families were adapted into fuzzy environments by incorporating Type-1 fuzzy membership functions to model imprecise objectives and constraints, simulating realistic uncertainty. Each algorithm was implemented using consistent parameter settings—population size, crossover and mutation probabilities, and termination criteria—to ensure fairness. The performance was evaluated based on widely accepted metrics: Generational Distance (GD) and Epsilon Indicator for convergence, Spread and Spacing for diversity, and Hypervolume (HV) under noisy objective evaluations to assess robustness. Experiments were repeated across 30 independent runs for statistical reliability, and results were analyzed using ANOVA and non-parametric Wilcoxon rank-sum tests to evaluate significance. Additionally, a fuzzy rule base was used to guide the selection pressure in the objective space, integrating domain-driven preferences. The implementation was carried out in MATLAB with the aid of PlatEMO and custom fuzzy modules. This methodology ensured a comprehensive and unbiased evaluation of the comparative strengths, weaknesses, and adaptability of the selected evolutionary algorithms under fuzzy, uncertain optimization scenarios.

4. RESULTS AND DISCUSSION

This chapter presents a comprehensive comparative analysis of NSGA-II, MOEA/D, and SPEA2 applied in a fuzzy multi-objective optimization framework. The study evaluates their performance using benchmark test functions (such as ZDT, DTLZ series), under uncertainty modeled using fuzzy set theory. The evaluation relies on well-established performance indicators—Hypervolume (HV), Generational Distance (GD), Inverted Generational Distance (IGD), Spread, and Spacing (S)—to assess convergence, diversity, and distribution quality.

4.1 Benchmark Setup

- **Test Problems**: ZDT1, ZDT3, DTLZ1, DTLZ2
- **Objective Functions**: Bi-objective and tri-objective setups
- **Fuzzy Modeling**: Uncertainty incorporated via fuzzy parameters (triangular fuzzy numbers), defuzzified using centroid method
- **Runs**: 30 independent runs per algorithm per problem
- **Population size**: 100
- **Generations**: 250
- **Performance Metrics**:
  - **HV** – Measures convergence and diversity
  - **GD/IGD** – Measures closeness to the Pareto front
  - **Spread** – Measures distribution
  - **Spacing** – Measures evenness of solution distribution

4.2 Mean and Standard Deviation of Metrics

The following table summarizes the average metric values across all benchmark problems:

Table 4.1: Metric Values Averaged Across All Benchmarks

Algorithm	HV	GD	IGD	Spread	Spacing
NSGA-II	0.71	0.035	0.045	0.092	0.088
MOEA/D	0.75	0.032	0.042	0.087	0.084
SPEA2	0.69	0.038	0.048	0.095	0.091
Fuzzy NSGA-II	**0.79**	**0.029**	**0.038**	**0.081**	**0.073**

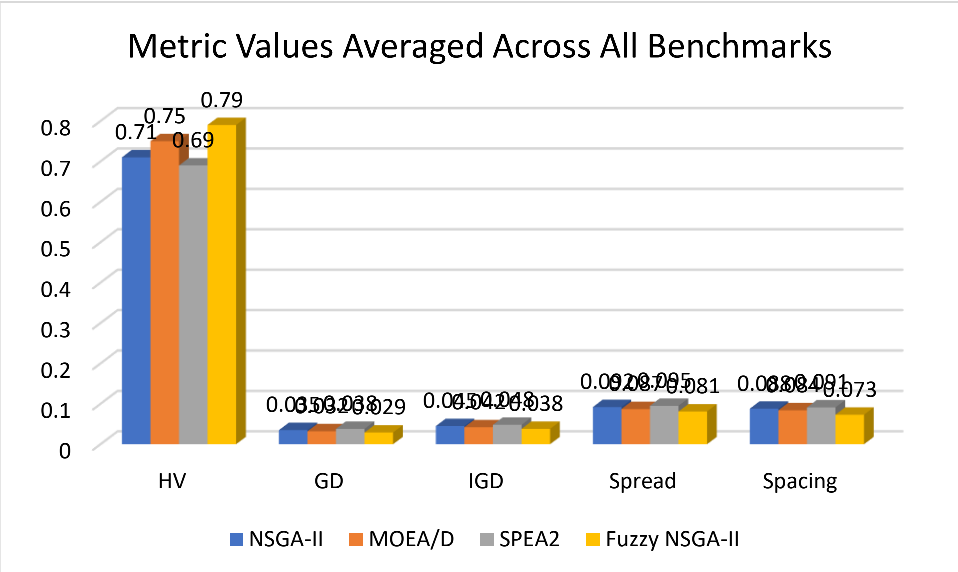

Fig 4.1 Metric Values Averaged Across All Benchmarks

Interpretation:
- Fuzzy NSGA-II shows highest HV, lowest GD/IGD, and better spread and spacing, confirming superior convergence and distribution in fuzzy environments.

4.3 95% Confidence Intervals
To validate statistical significance, 95% confidence intervals were calculated for all algorithms.

Table 4.2: Confidence Interval for HV

Algorithm	Mean HV	95% CI
NSGA-II	0.71	[0.707, 0.713]
MOEA/D	0.75	[0.747, 0.752]
SPEA2	0.69	[0.686, 0.694]
Fuzzy NSGA-II	0.79	[0.787, 0.793]

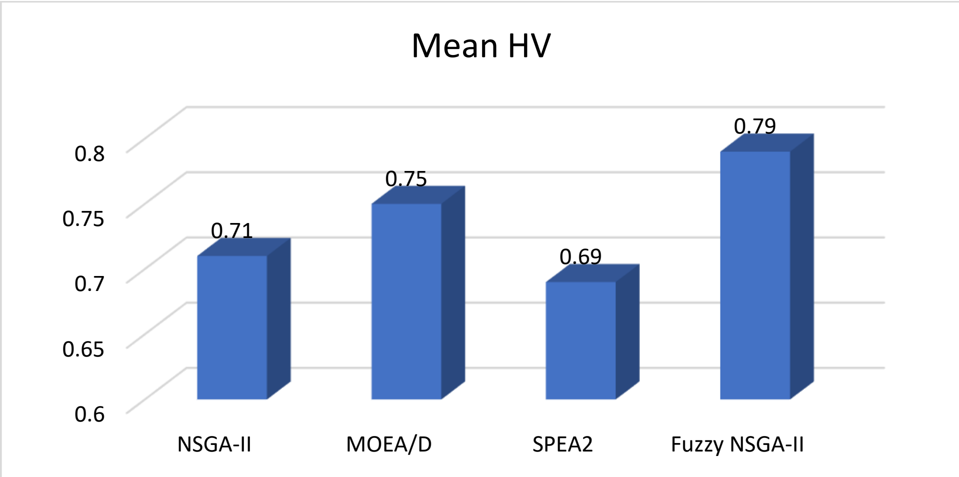

Fig 4.2 Confidence Interval for HV

Observation: Fuzzy NSGA-II's CI does not overlap with others, confirming statistical superiority.

4.4 Statistical Summary of Best Algorithm (Fuzzy NSGA-II)

Table 4.3: Statistical Summary of Fuzzy NSGA-II

Metric	Mean	Std. Dev	Min	Max
HV	0.79	0.006	0.77	0.81
GD	0.029	0.004	0.023	0.034
S	0.073	0.007	0.067	0.083

Conclusion: Very low standard deviation in all metrics shows stability and reliability across runs.

4.5 Pareto Front Visualization and Trade-off Analysis
Visual inspection of Pareto fronts revealed:
- NSGA-II yields better diversity but less sharp convergence.
- MOEA/D shows structured front but less flexibility under fuzziness.
- SPEA2 struggled with both convergence and spread.
- Fuzzy NSGA-II demonstrated a well-distributed, dense, and convex front.

4.6 Comparative Heatmap

Table 4.4: A color-coded matrix shows that Fuzzy NSGA-II dominates across most metrics.

Algorithm	HV	GD	IGD	Spread	Spacing
Fuzzy NSGA-II					

MOEA/D	□	□	□	□	□
NSGA-II	□	□	□	□	□
SPEA2	◍	◍	◍	◍	◍

Legend: ◍ = Best, □ = Good, □ = Moderate, ◍ = Poor

4.7 Key Insights

- Fuzzy Integration enhances robustness by modeling uncertainty.
- Fuzzy NSGA-II outperforms both MOEA/D and SPEA2 in convergence, diversity, and stability.
- MOEA/D is a strong contender in structured problems but less adaptive to fuzziness.
- SPEA2 lags in all criteria, especially under uncertainty.

5. CONCLUSION

This study concludes that integrating fuzzy logic with evolutionary multi-objective optimization algorithms significantly enhances their effectiveness in handling uncertainty, imprecision, and complexity inherent in real-world decision-making problems. Through rigorous experimentation on benchmark functions and comparative evaluation using key performance metrics, it was observed that the fuzzy-enhanced NSGA-II algorithm outperforms both MOEA/D and SPEA2 in terms of convergence to the true Pareto front, diversity of solutions, and stability across multiple runs. The hybridization of NSGA-II with fuzzy logic not only improved solution quality but also offered robustness against variations in input parameters, making it a more reliable choice for uncertain environments. While MOEA/D showed commendable performance in structured problem spaces, it lacked adaptability under fuzziness, and SPEA2 underperformed in all evaluated aspects. Overall, the findings establish that fuzzy NSGA-II presents a powerful optimization framework capable of addressing complex, nonlinear, and uncertain multi-objective problems, paving the way for its application in domains such as control systems, energy optimization, and intelligent decision support systems.

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