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THz Radiation in Longitudinally Polarized Undulator

Mukesh Kumar

University School of Basic and Applied Sciences, GGSIP University, New Delhi-110078, India
laspimk@ipu.ac.in

Abstract

In this article, numerical and analytical analysis of electron trajectories for brightness in THz region of longitudinal undulator is numerically studied. Pondermotive force and stimulated emission is investigated in THz radiation. Spectral brightness, average power of THz radiation emitted by longitudinal undulator is analyzed numerically.

Keywords: Longitudinal Undulator**,** Electron Trajectories, Pondermotive Force and Spectral Brightness.

1. Introduction

Free Electron Lasers (FELs) have widely recognized importance over quantum lasers in terms of widespread tunability from microwave to coherent x-rays including THz radiations, high spectral brightness and precised monochromaticity [1]. Tunability is achieved either by undulator parameter K indirectly varying magnetic field of undulator or controlling electron beam energy generated from linear accelerator. In quantum Lasers, the evolution of high intensity Laser beam may potentially damage the transparent medium of solid-state Laser oscillator. When a Laser light is propagated through a series of optical amplifier for the development of high intensity, Laser medium albeit a considerable amount of energy is dissipated by the medium leading to a rise in temperature causing thermal stress within the material [2-3].

A Terahertz (THz) radiation source with better spatial coherence, high spectral brightness and wider tunability requirement is multiplied amid various advanced applications in medical imaging, THz spectroscopy, defense applications, scientific research and commercial industry [4-7]. The present work describes electron trajectories studied analytically using Generalised Bessel Functions (GBFs) [8-9]. Longitudinal Pondermotive force responsible for microbunching is derived and investigated using relativistic Lorentz force equation [10-12]. The expression for on axis and off axis spectral brightness in case of longitudinally polarized undulator is investigated. Numerical calculations in this article confirms the production of THz radiation with high spectral brightness with longitudinally polarized undulator [13-14].

Magnetic Field of Longitudinal Undulator

Magnetic field of longitudinally polarized undulator is an integral part of Free Electron Laser development and is responsible for production of coherent THz radiation over wide range of spectrum. The field is given by $\vec{B} = 0.\hat{x} + 0.\hat{y} + B_0 \sin(k_u z).\hat{z}$, where B_0 is the amplitude of the undulator field [15]. The amplitude B_0 of a longitudinal undulator magnetic field.

Electron Trajectories

We use a Relativistic Lorentz Force (RLF) equation $d\vec{p}/dt = -ec(\vec{\beta} \times \vec{B})$ (1)

where p is oscillatory momentum gained by electron beam in transverse directions within undulator, B is the longitudinal undulator field, β is normalized velocity of electron, for the calculations of velocity and electron trajectories using Generalised Bessel Function (GBF) and Relativistic Lorentz Force (RLF) equation will derive normalized velocities in x and y directions for I_b beam current comprising of N_e electrons (cf (14)).

On simplification, electron trajectories are defined as

$$x/c = x_0/c + \Re \sum_{n=-\infty}^{n=\infty} \left[\sin\{n(\omega_u \beta_z t - \pi/2)\} - \sin\{-n\pi/2\} \right] \tag{2}$$

$$\Re = \{\beta_\perp / n\omega_u \beta_z\} J_n \left\{ 2\pi N_u I_b K / e\gamma\omega_u \beta_z^2 \right\}$$

$$y/c = y_0/c + \Re \sum_{n=-\infty}^{n=\infty} \left[\cos\{n(\omega_u \beta_z t - \pi/2)\} - \cos\{-n\pi/2\} \right] \tag{3}$$

Where $\beta_z = 1 - \frac{1}{2\gamma^2} - \frac{\beta_\perp^2}{2} J_n^2 (2\pi N_u I_b K / e\gamma\omega_u \beta_z^2)$ (4)

When a beam of N_e electrons constituting current I_b enters an undulator field with an initial transverse momentum, its trajectory exhibits periodic modulations that reflect multiple harmonic components of its motion [16].

Pondermotive Force

The Relativistic Lorentz Force (RLF) equation is given by

$$\vec{F}_z = -ec(\vec{\beta}_\perp \times \vec{B}_L) \tag{5}$$

where B_L is magnetic field of radiation, $\beta_\perp$ is transverse velocity of electrons. $\vec{B}_L = B_{0L} \cos\{\psi\}.\hat{x} + B_{0L} \sin\{\psi\}.\hat{y}$, is left-handed circularly polarized radiation field where $\psi = (n_1 k_1 z - n_1 \omega_1 t + \phi_L)$, and n_1 represents the emission harmonics. Hence the expression for

But real time Free Electron Lasers (FELs) employs an electron beam with I_b beam current comprising of N_e electrons produces pondermotive force [39] is given by

$$F_z = -2\pi \left(\frac{E_{0L} \beta_\perp I_b N_u}{\omega_u \beta_z} \right) \sum_{n=\infty}^{n=-\infty} J_n (2\pi N_u I_b K / e\gamma\omega_u \beta_z^2) \sin\{n_1 (k_u + k_1) z - n_1 \omega_1 t + \phi_L\} \tag{6}$$

Where E_{0L} is the amplitude of radiation, I_b is the beam current, N_u is the number of undulator periods and $\psi_s = \{n_1(k_u + k_1)z - n_1\omega_1 t + \phi_L\}$.

Brightness of Undulator:

The expression of undulator brightness for N_e electrons in a bunch is given:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega_L^2}{16\pi^3\varepsilon_0 c}\left[N_e + N_e(N_e-1)f(\omega)\right]\left|\int_{-\infty}^{+\infty}\left[\vec{n}\times(\vec{n}\times\vec{\beta})\right]\exp\{i\omega_L(t-\tfrac{\vec{n}.\vec{r}}{c})\}dt\right|^2 \qquad (7)$$

where $f(\omega) = \left|\int \exp(i\omega r/c)S(\vec{r})d^3r\right|^2$ is the form factor responsible for coherent enhancement of radiation for N_e electrons, $S(\vec{r})$ is a steady normalised particle distribution function of electron cluster. The detail expression is given as

$$\tilde{\tilde{B}}(d\dot{N}/d\Omega, d\omega/\omega, 0.1\%BW) = \sum_{m=1}^{\infty}\frac{eN_u^2\gamma^4 I_b(A)}{\pi\varepsilon_0\hbar c\sigma_x(mm)\sigma_y(mm)\sqrt{1+\frac{\sigma_x'^2}{\theta_{cen}^2}}\sqrt{1+\frac{\sigma_y'^2}{\theta_{cen}^2}}}\frac{m^2\sin c^2(\nu_m/2)}{[1+\gamma^2\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)+\gamma^2\psi^2]^3}$$
$$\times\Big[0.5\gamma^2\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)\{J_{m-n}^2(m\eta_1, m\eta_2)+J_{m+n}^2(m\eta_1, m\eta_2)\}-(\gamma\psi)(\gamma\beta_\perp)J_n(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)$$
$$\times\beta_z J_m(m\eta_1,\ m\eta_2)\{J_{m-n}(m\eta_1, m\eta_2)+J_{m+n}(m\eta_1, m\eta_2)\}$$
$$+\gamma^2\psi^2\{\beta_z^2 J_m^2(m\eta_1, m\eta_2)-0.25\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)((J_{m-n}(m\eta_1, m\eta_2)$$
$$+J_{m+n}(m\eta_1, m\eta_2))^2\}\Big]$$

(8)

On simplification, we get the modified expression

$$\tilde{\tilde{B}}(d\dot{N}/d\Omega, d\omega/\omega, 0.1\%BW) = \sum_{m=1}^{\infty}\frac{2.91\times10^6\gamma^4 N_u^2 I_b(A)}{\sigma_x(mm)\sigma_y(mm)\sqrt{1+\frac{\sigma_x'^2}{\theta_{cen}^2}}\sqrt{1+\frac{\sigma_y'^2}{\theta_{cen}^2}}}\frac{m^2\sin c^2(\nu_m/2)}{[1+\gamma^2\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)+\gamma^2\psi^2]^3}$$
$$\times\Big[0.5\gamma^2\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)\{J_{m-n}^2(m\eta_1, m\eta_2)\}-(\gamma\psi)(\gamma\beta_\perp)J_n(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)$$
$$\times\beta_z J_m(m\eta_1,\ m\eta_2)\{J_{m-n}(m\eta_1, m\eta_2)+J_{m+n}(m\eta_1, m\eta_2)\}$$
$$+\gamma^2\psi^2\{\beta_z^2 J_m^2(m\eta_1, m\eta_2)-0.25\beta_\perp^2 J_n^2(2\pi N_u I_b K/e\gamma\omega_u\beta_z^2)((J_{m-n}(m\eta_1, m\eta_2)$$
$$+J_{m+n}(m\eta_1, m\eta_2))^2\}\Big]\left[\frac{Photons/sec}{mm^2 mrad^2(0.1\%BW)}\right]$$

(9)

Numerical Results and Discussion

The brightness of LU is derived using Lienard-Wiechert expression. The results so obtained demonstrate that the LU introduces distinctive electron trajectories due to the LU field configuration, resulting in unique radiation patterns. Moreover, the final off axis expression figured out that the spectral brightness of the longitudinal undulator field is azimuthally independent.

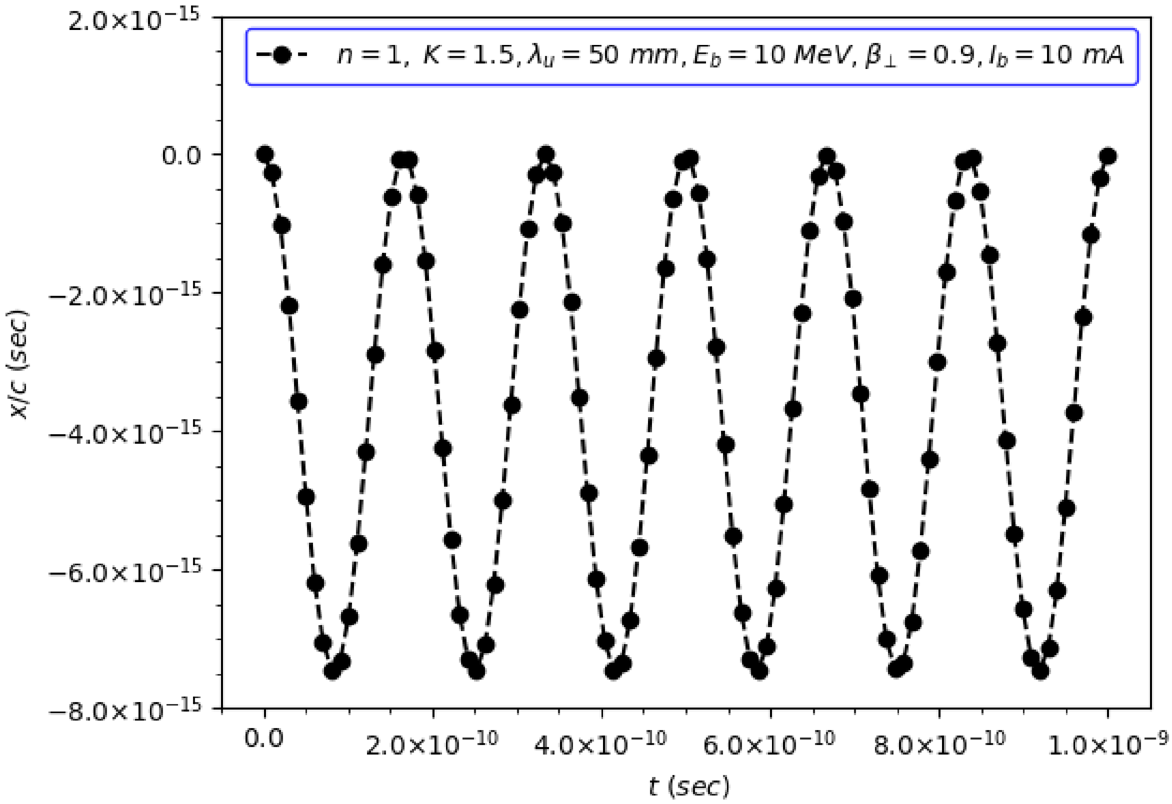

Fig.1 Electron trajectory for fundamental harmonic n=1 with K=1.5, $\lambda_u = 50$ mm, electron beam energy E_b=10 MeV and beam current I_b=10 mA.

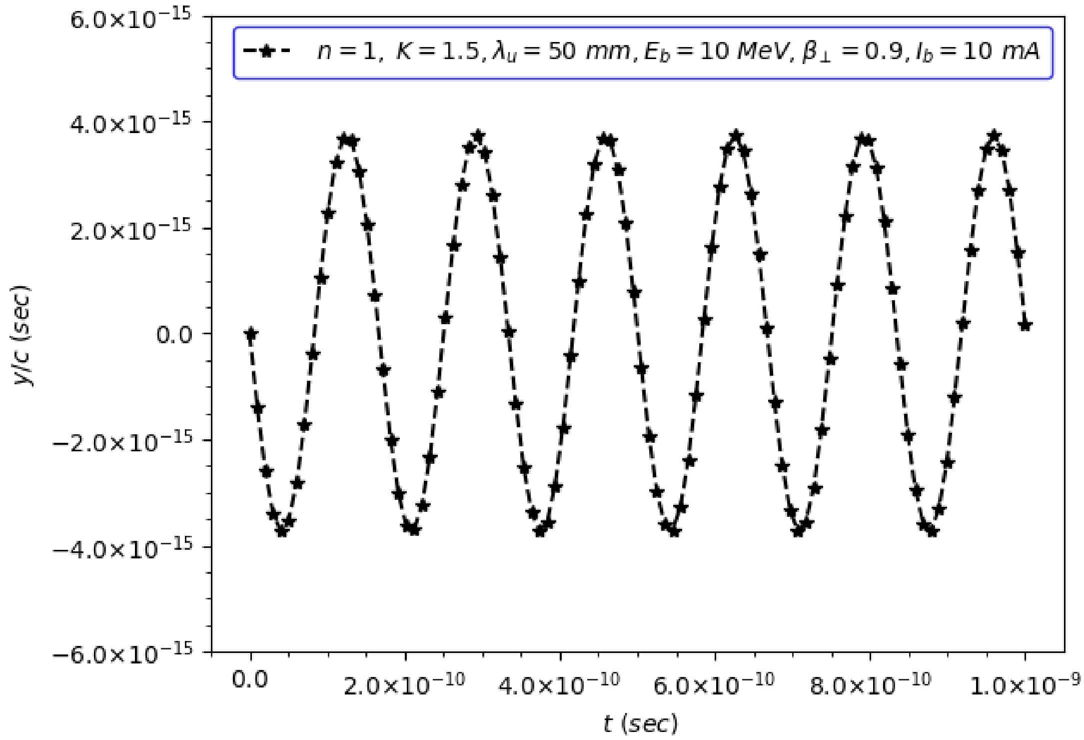

Fig.2 Electron trajectory for fundamental harmonic n=1 with K=1.5, $\lambda_u = 50$ mm, electron beam energy E_b=10 MeV and beam current I_b=10 mA.

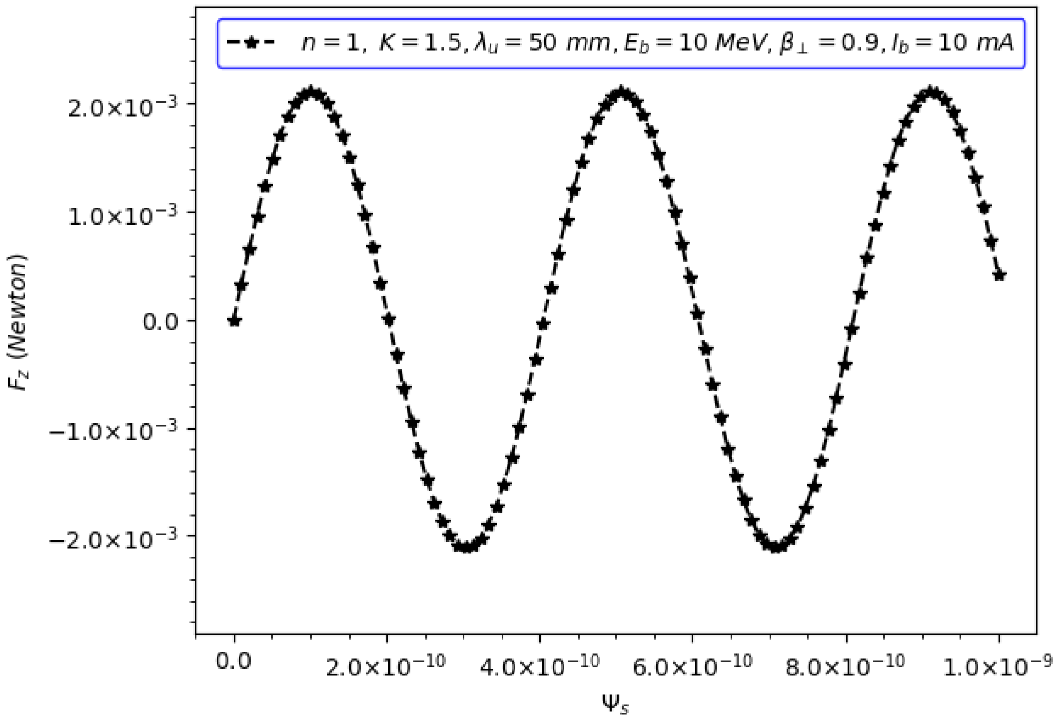

Fig.3 Pondermotive force F_z vs phase ψ_s for fundamental harmonic n=1 with K=1.5, $\lambda_u = 50$ mm, electron beam energy E_b=10 MeV and beam current I_b=10 mA.

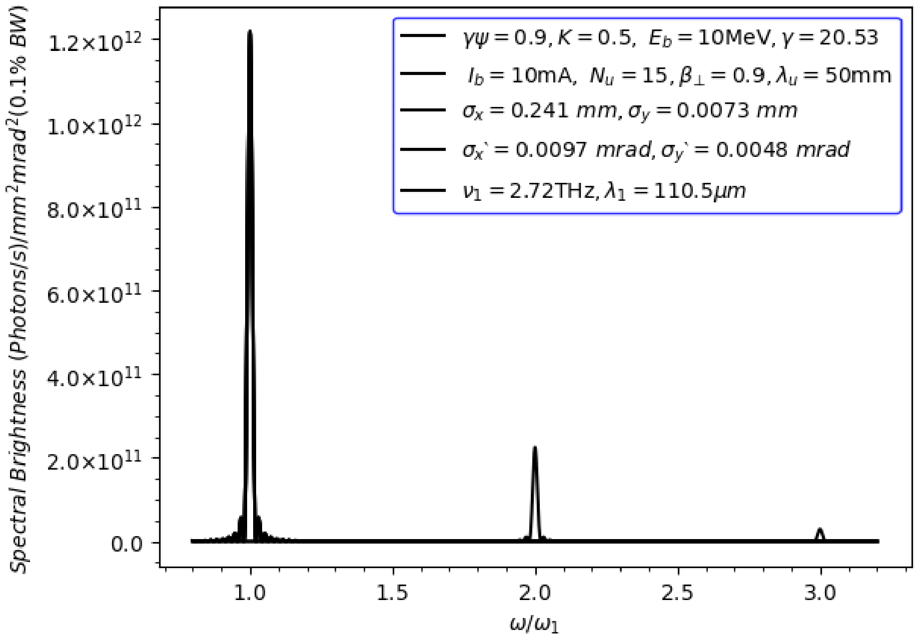

Fig.4 Spectral brightness of longitudinal undulator as a function of $\gamma\psi$ with E_b=10 MeV.

Conclusion

In this study, with relevant electron beam parameters, we conducted a comprehensive investigation numerically into the pondermotive force, off-axis spectral brightness, and average power associated with Terahertz (THz) radiation as produced by Longitudinal Undulator Free Electron Laser (LUFEL). A significant observation is the dependence of the amplitude of spectral brightness on the energy factor γ. This key finding suggests that LU-based FEL systems have the potential to generate considerably higher spectral brightness compared to conventional FEL methods.

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Conflicts of interest

The corresponding authors declared that there are no conflicts of interest in this manuscript.

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